An Alternative Analytic Beach Nourishment Planform Model

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PRELUDE

Observation from Town of Palm Beach (Dean, 1998)

"Somebody should derive an analytic planform solution that is not symmetric about the initial nourishment centerline."

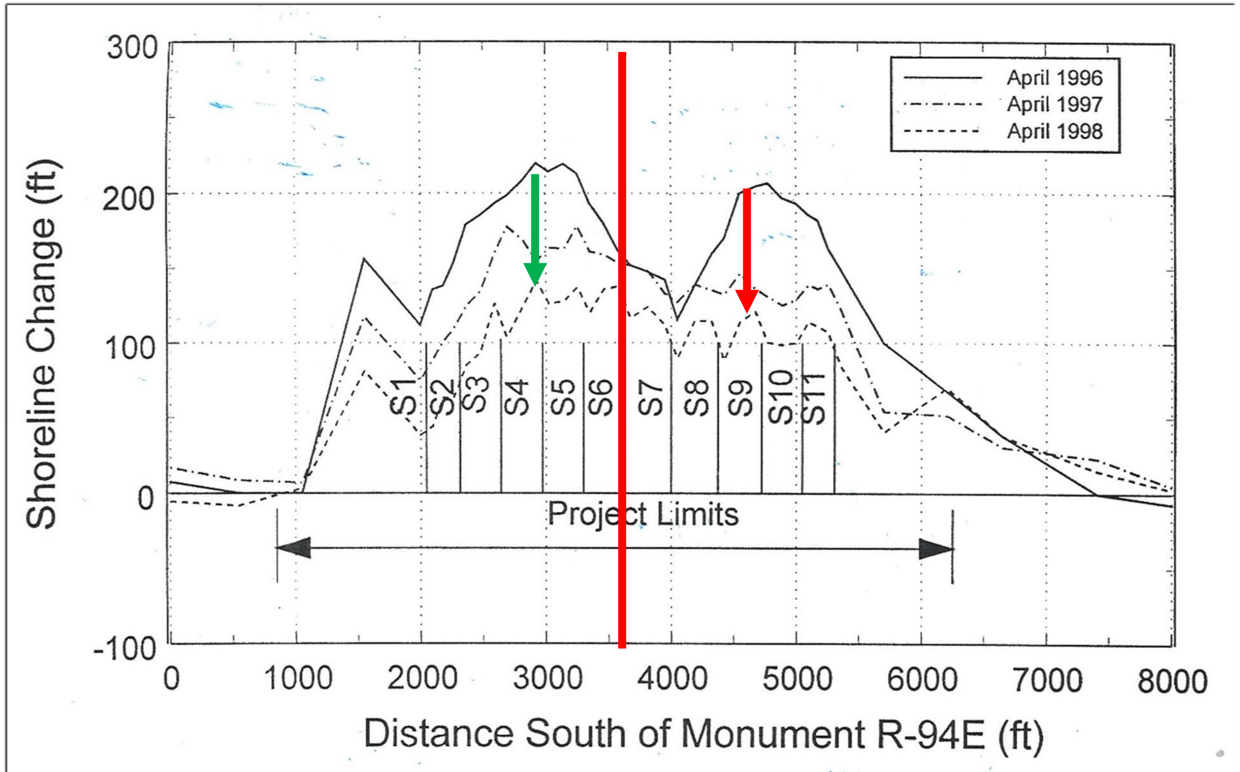


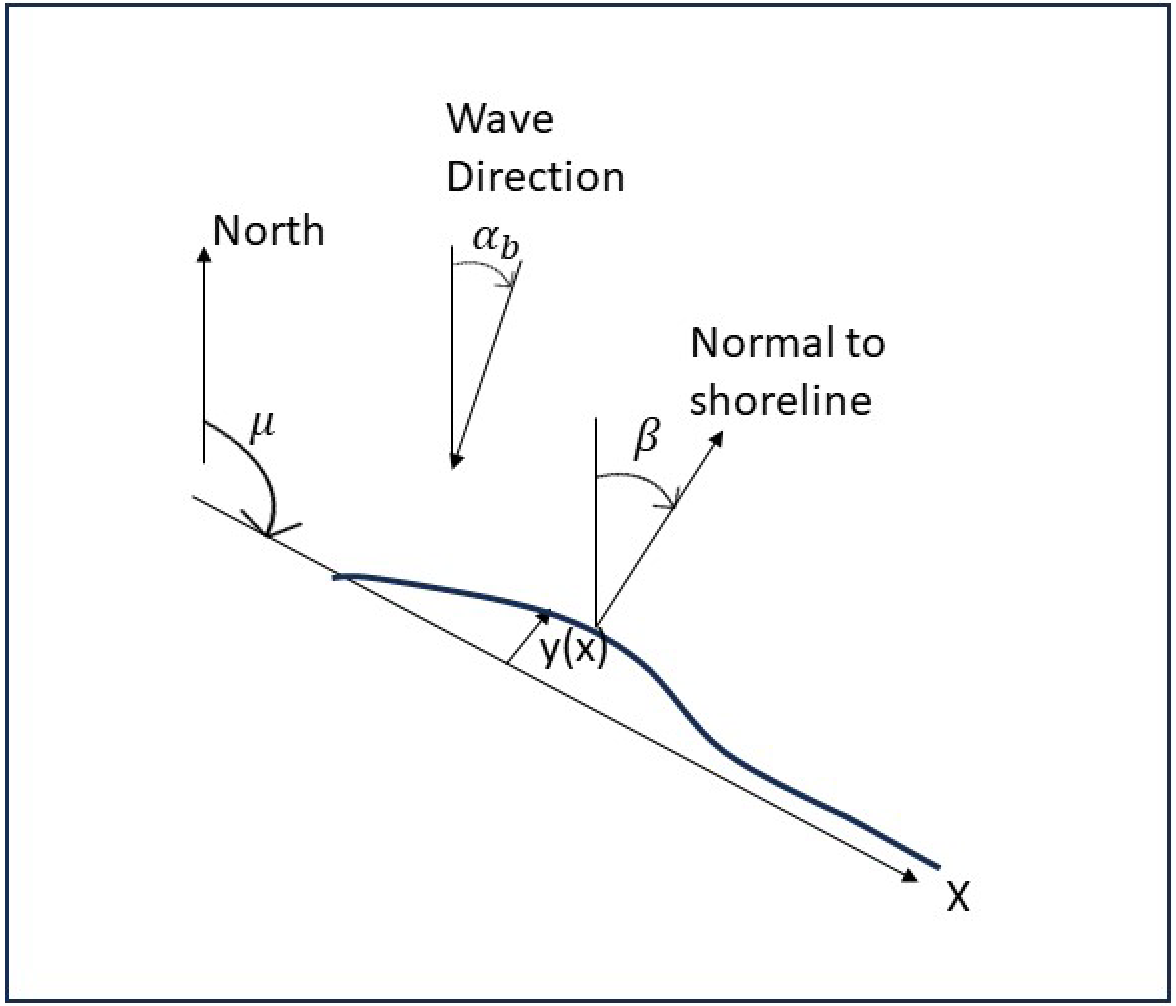
Figure 10. NGVD Shoreline Changes Due to Nourishment Project

WHY BEACH NOURISHMENT MODELS?

- We seek analytic models to understand the principal coastal processes. They can be used for coastal processes education.
- We can use analytic models to verify numerical models of similar derivation.
- Analytic models are computationally efficient.

- There are many (good) models to choose from. There are more sophisticated numerical models which can utilize more complex wave, tide, and wind forcing and resulting sediment transports.

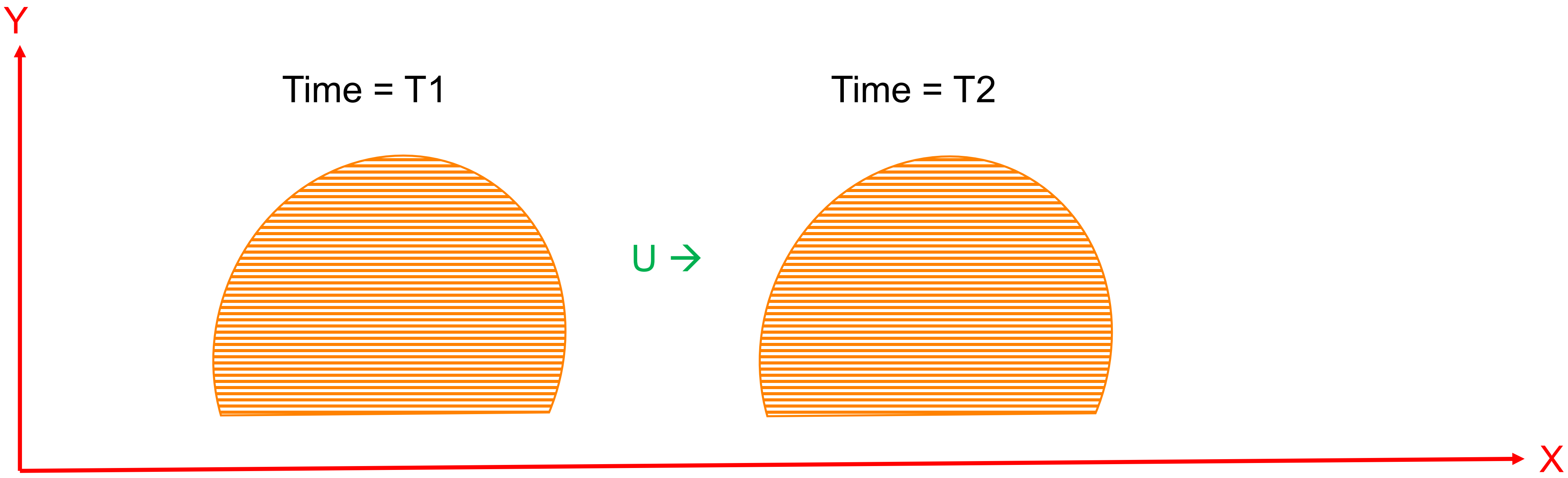
PROBLEM DEFINITION SKETCH



COMPARISON OF MODEL DEVELOPMENT

Item	Diffusion Model
Principle	Conservation of Sand (Mass)
Author	Pelnard-Considere (1956)
Mathematical Relationship	$\frac{dy}{dt} = G\frac{d^2y}{dx^2}$

PLANFORM ADVECTION



COMPARISON OF MODEL DEVELOPMENT

Item	Diffusion Model	Advective Diffusion Model
Principle	Conservation of Sand (Mass)	Conservation of Sediment Flux
Author	Pelnard-Considere (1956)	Inman, D. (1987)
Mathematical Relationship	$\frac{\mathrm{dy}}{\mathrm{dt}} = G\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}$	$\frac{\mathrm{dy}}{\mathrm{dt}} = G\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}$ - U $\frac{\mathrm{dy}}{\mathrm{dx}}$
Solution Techniques	Separation variables (Greenberg, 1978); Laplace transforms (Larson, Hanson and, Kraus (1987)	
Analytic solution for rectangular planform of width Y	$y(x,t) = \frac{Y}{2}\left(\mathrm{erf}\left(\frac{L\left(\frac{2x}{L}+1\right)}{\sqrt{\frac{4}{Gt}}}\right) - \mathrm{erf}\left(\frac{L\left(\frac{2x}{L}-1\right)}{\sqrt{\left(\frac{4}{Gt}\right)}}\right)\right)$	
Author	Dean (2002), Larson Hanson and Kraus (1987), others	

SOLUTION TO ADVECTIVE DIFFUSION EQUATION

- Outlined approach of Larson and Kraus (1991),
- Detailed class notes from Cushman-Roisin (2012),

The advective diffusion equation, for constant advection velocity U, is solved by a change in variables:

$$y^* = y$$
$$x^* = x - Ut$$
$$t^* = t$$

COMPARISON OF MODEL DEVELOPMENT

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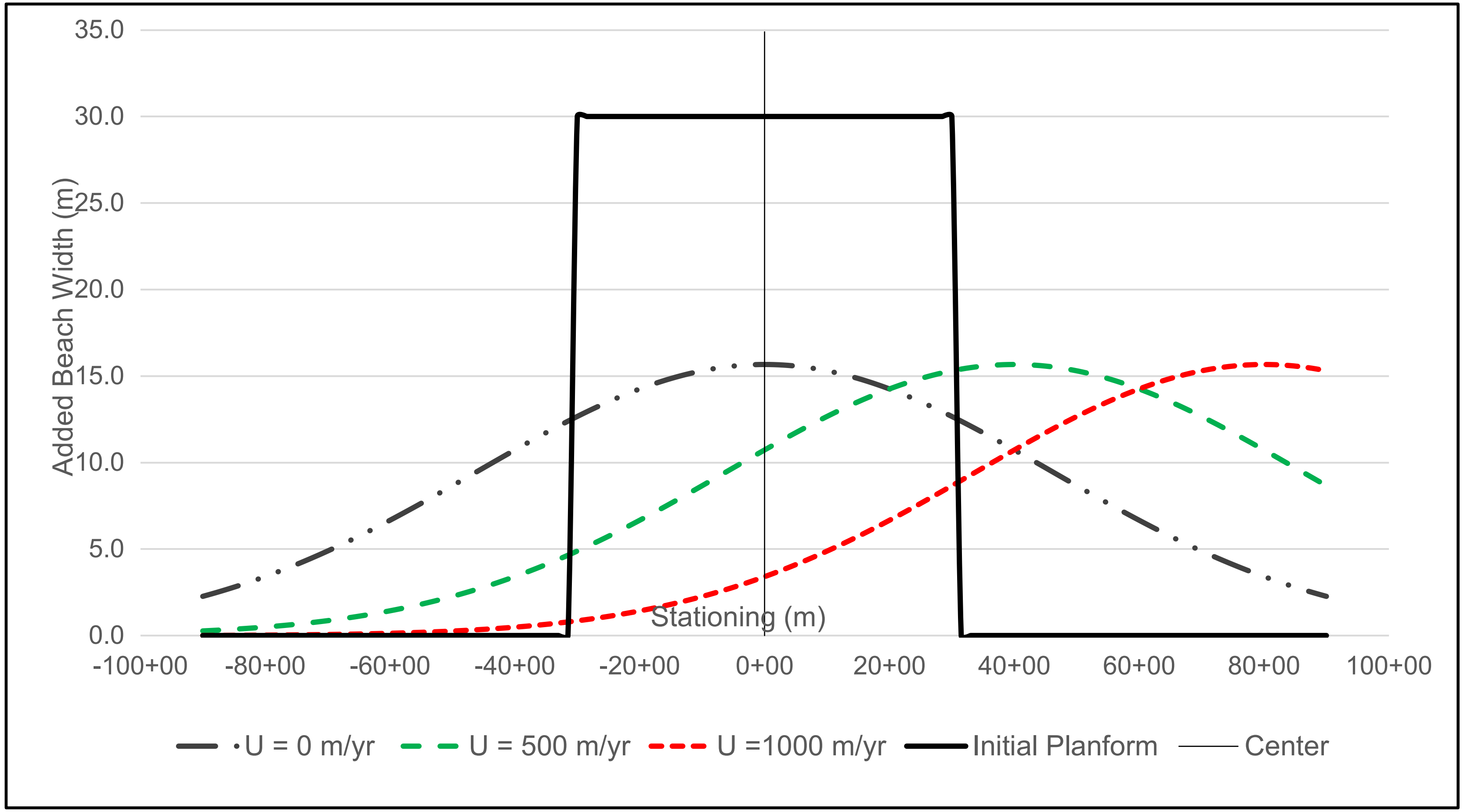
Mathematical Relationship	$\frac{dy}{dt} = G\frac{d^2y}{dx^2}$	$\frac{dy}{dt} = G\frac{d^2y}{dx^2} - U\frac{dy}{dx}$
Solution Techniques	Separation variables (Greenberg, 1978); Laplace transforms (Larson, Hanson and, Kraus (1987)	Larson and Kraus (1991) outlined an approach using a change in variables. Cushman-Roisin (2012) provides detailed solution.
Analytic solution for rectangular planform of width Y	$y(x,t) = \frac{Y}{2}\left(\mathrm{erf}\left(\frac{L\left(\frac{2x}{L}+1\right)}{\sqrt{\frac{4}{Gt}}}\right) - \mathrm{erf}\left(\frac{L\left(\frac{2x}{L}-1\right)}{\sqrt{\left(\frac{4}{Gt}\right)}}\right)\right)$	$y(x,t) = \frac{Y}{2}\left(\mathrm{erf}\left(\frac{L\left(\frac{2(x-Ut)}{L}+1\right)}{\sqrt{\frac{4}{Gt}}}\right) - \mathrm{erf}\left(\frac{L\left(\frac{2(x-Ut)}{L}-1\right)}{\sqrt{\left(\frac{4}{Gt}\right)}}\right)\right)$
Author	Dean (2002), Larson Hanson and Kraus (1987), others	Mann (in press)

COMPARISON OF PLANFORM EVOLUTION FOR SELECT VELOCITIES

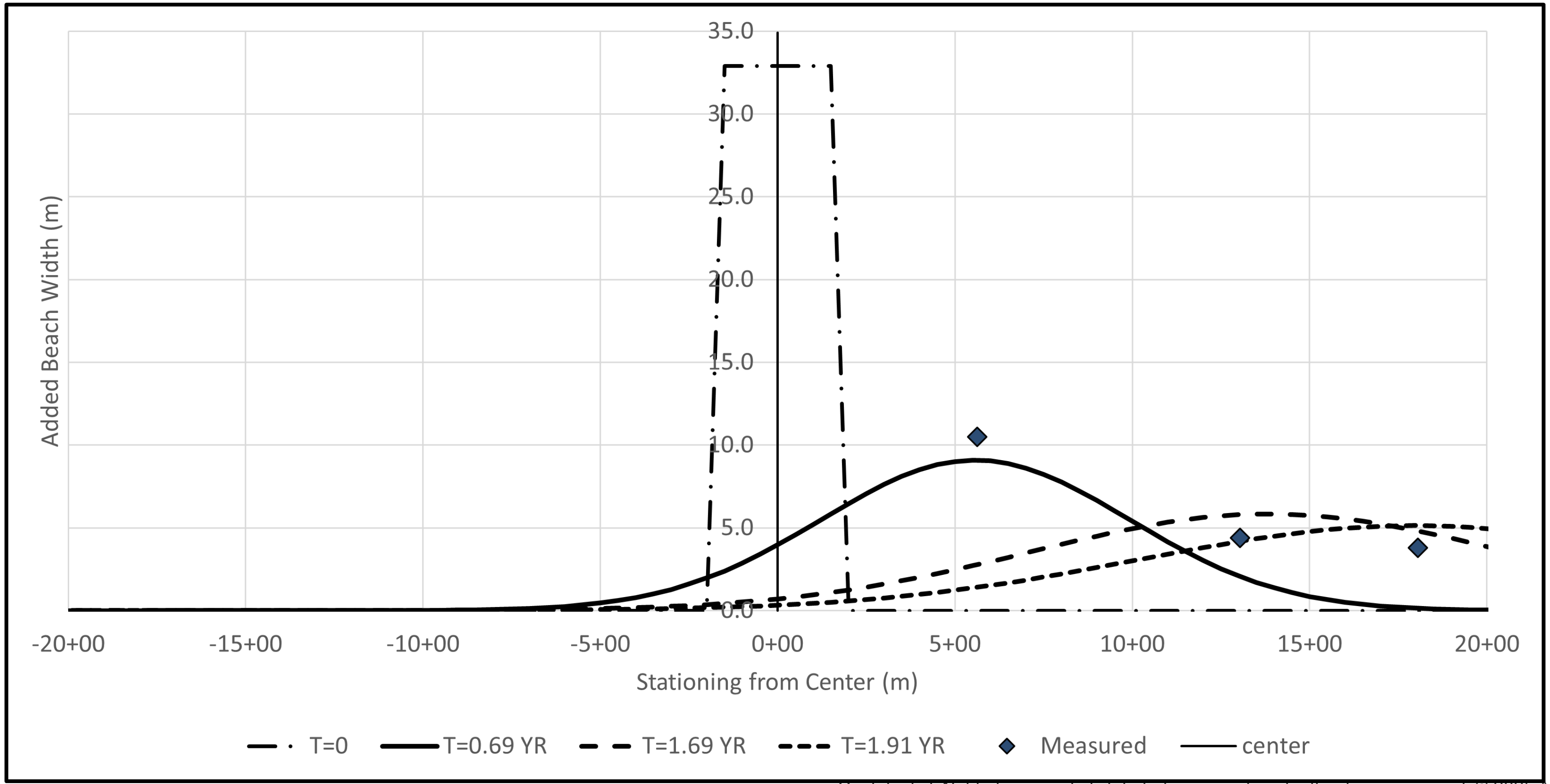
Y= 30 m, L = 6000 m, t = 8 years, G = 0.0354 m^2/s



DEMONSTRATION 1: GROVE, SONU, AND DYKSTRA (1987)

153,000 cubic meters of sand was released onto the San Onofre, CA shoreline.

– y(x,t) for U = 820 m/yr and G = 0.0036 m^2/s



REAL BEACH NOURISHMENTS

- Shorelines are not initially straight.
- Background erosion exists and is not uniform.
- There can be sediment differences between the native sands and nourished sands.
- Storm induced cross shore movements occur (non-uniformly).

- Therefore, comparing a shoreline planform to an asbuilt nourishment performance must be done cautiously.

DEMONSTRATION 2: VANDERBILT BEACH FL

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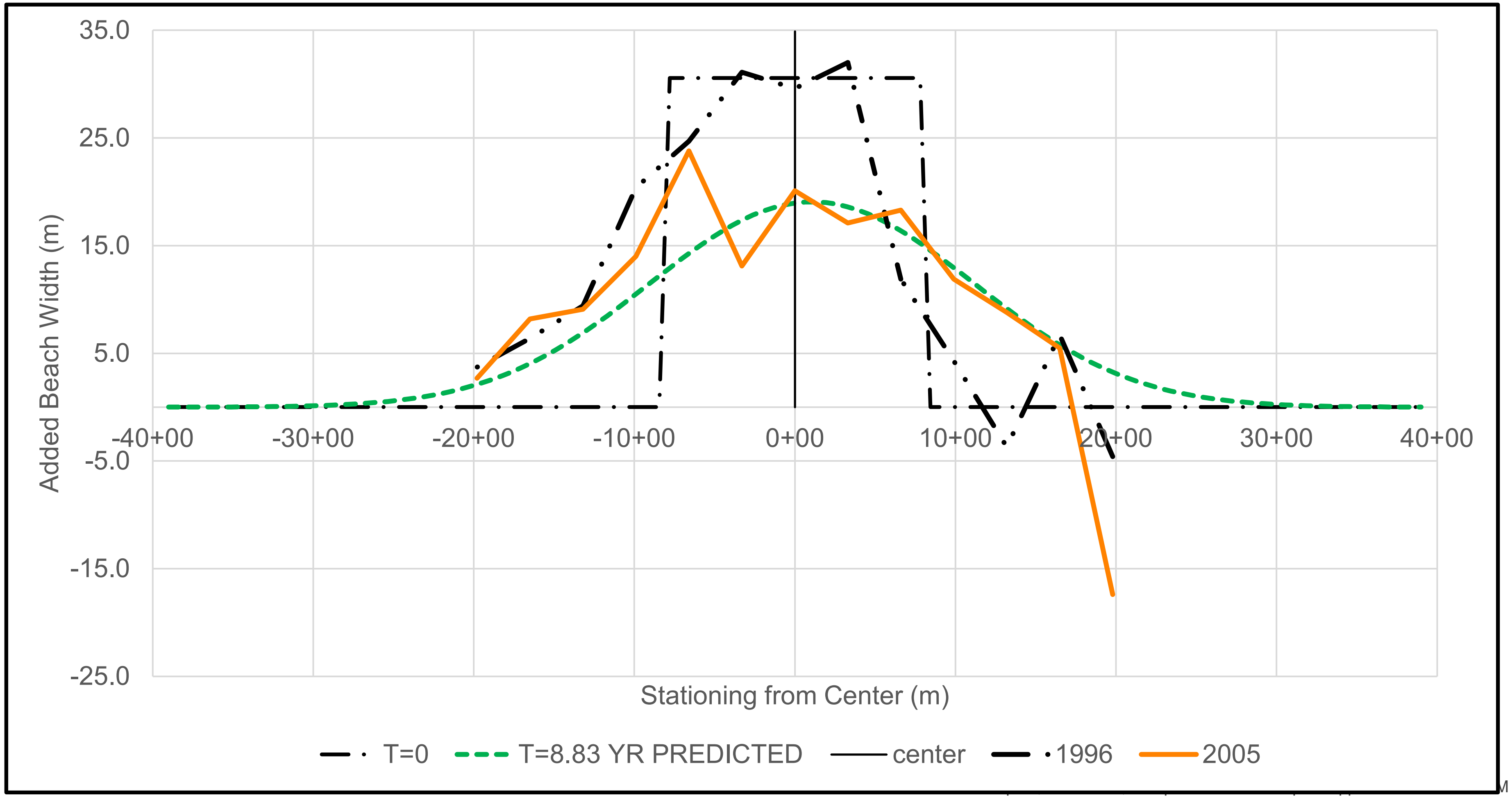
DEMONSTRATION 2: VANDERBILT BEACH FL, 1996

U = 12 m/yr, G = 0.0014 m²/s, L=1560 m, t=8.83 yr



ADVECTIVE VELOCITY LIMITATIONS

- George Box (1976) said, “Since all models are wrong, the scientist must be alert to what is importantly wrong.”
- 1. On any given day, the beach nourishment (planform) does not actually move at a constant rate U, it is likely episodic.
- 2. As we are modeling the planform (shoreline), the cross-shore distribution of the advective velocity (from the dune toe to the depth of closure) is not likely uniform. The advective velocity at the shoreline is, at best, an estimate of the cross shore distribution of the advective velocity.
- 3. There is little consensus on how to predict U:

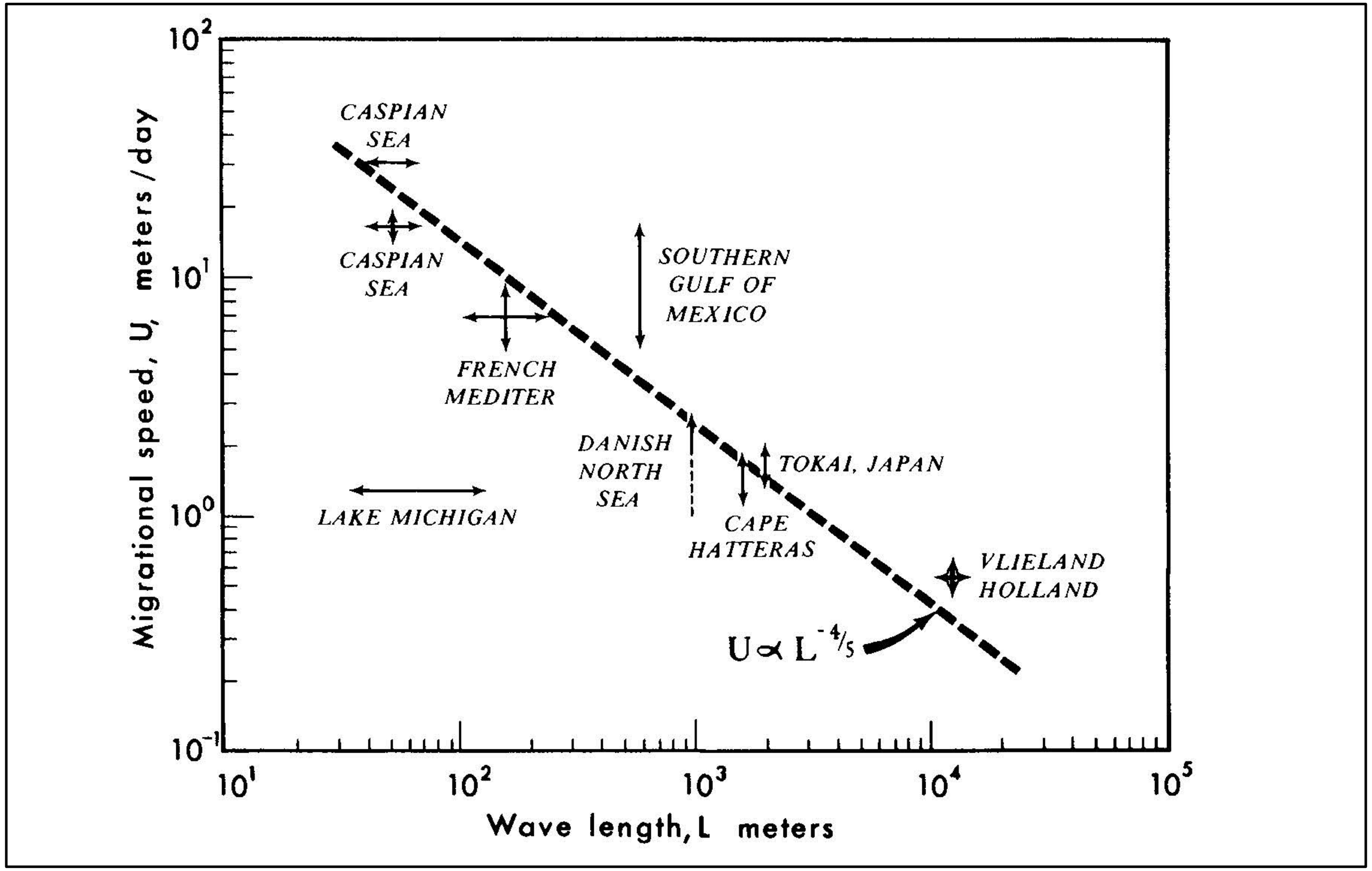
ESTIMATION OF ADVECTIVE VELOCITY, U

– 1. Sonu (1968) studied the migration of alongshore sand waves, and suggests that

$$U\ [m/day] \sim L^{-0.8}\ [m], \text{ or}$$

$$U\ [m/day] \sim L^{-1}\ [m]$$

That is, shorter nourishment lengths will advect faster than longer nourishment lengths. This is in addition to the faster diffusion (as explained by Dean, 2002) !



ESTIMATION OF ADVECTIVE VELOCITY, U

– 2. Inman (1987) suggests that

$U = Q_{net}/v$,

Where
Q_{net} is the net alongshore sediment transport [m^3/yr]
v is the interannual unit volume of sand [m^3/m] that moves cross shore.

But, what if v trends toward zero?

ESTIMATION OF ADVECTIVE VELOCITY, U

– 3. Hanson, Thevenot, Kraus (1996) and Thevenot and Kraus (1995) relate U to the alongshore water discharge, R, (Kraus and J. Dean, 1987),

– $U = \alpha \frac{(R - R_{crit})}{y\,(h^{*} + B)}$

Where

- α = 8.8 x10 $^{-4}$
- R_{crit} = 2.4 m^3/s
- R = 0.5 d_b y_b v_{ls},
  - Where $v_{ls} = \frac{1.35}{2} k \sqrt{g d_b} \sin(2\theta_b)$ (Komar and Inman, 1970),
  - d_b and y_b are the depth of breaking and the distance offshore to breaking, respectively.

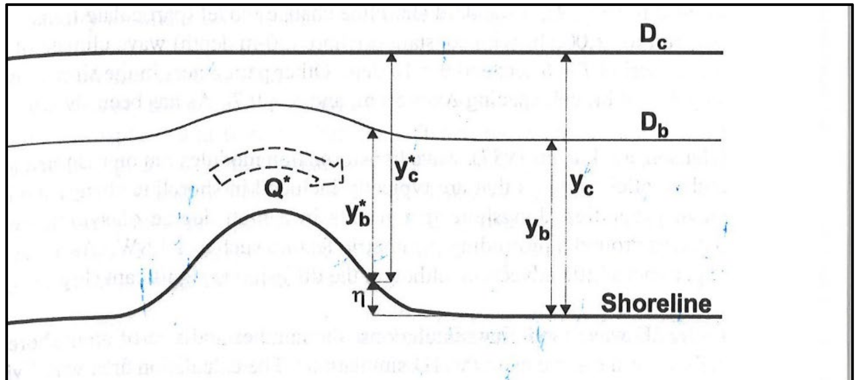


Fig 1. Definition sketch for the contracted surf zone approach.

But U = f(y(x)),and is no longer constant.

CONCLUSIONS

- An alternative beach planform model is proposed based on the advective diffusion equation.
  - Yields non symmetric solutions about the initial centerline of the planform.
  - Model provides a good (engineering) comparison to the Grove, Sonu, and Dykstra (1987) data set.
  - Combined with the results of Sonu (1968), shorter nourishment lengths are penalized with higher advection velocities in addition to higher rates of diffusion (after Dean (2002).

CONCLUSIONS

- Ability to accurately predict the advective velocity limits accurate prediction of beach nourishment planform evolution.
- Comparison of modelled planform evolutions to actual nourishments is challenging due to the model assumptions.
- Nevertheless, using the analytic model to compare engineering alternatives can be useful and insightful (after Roelvink and Reniers, 2012).

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ACKNOWLEDGEMENTS

- Pelnard-Considere
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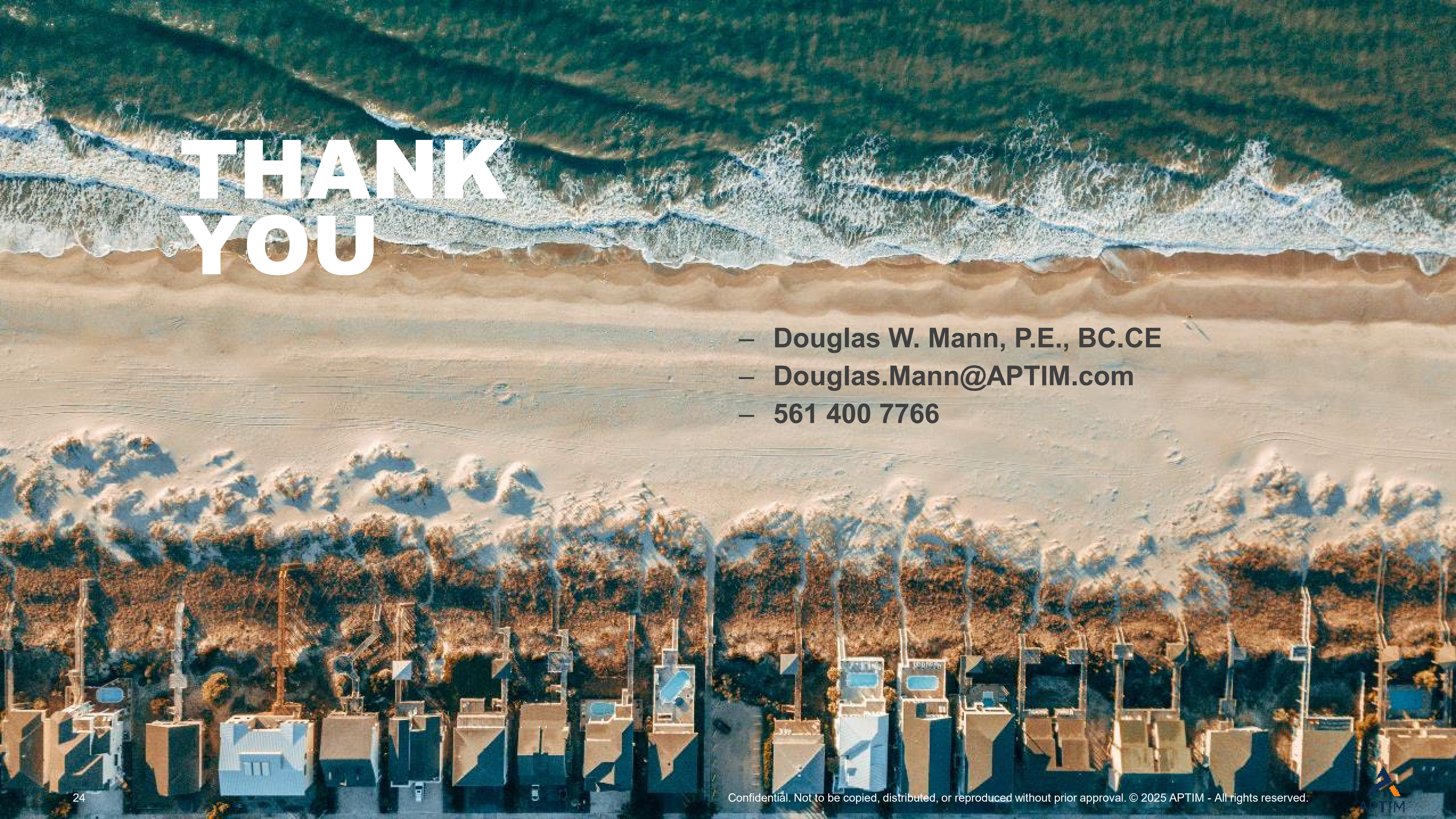
- Dean and Yoo
- Michael Greenberg
- Hanson, Thevenot, and Kraus
- Kraus and Dean
- Larson, Hanson, and Kraus
- Choule Sonu
- Huisman, De Vries, Walstra, Roelvink, Ranasinghe, and Stive
- George Box
- Jean Claude Eugene Peclet

APOLOGIES

- This work took way too long to complete.
- I have now removed a master's thesis topic from an inquiring young mind. But there is plenty to do regarding the advective velocity, U.



THANK YOU

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